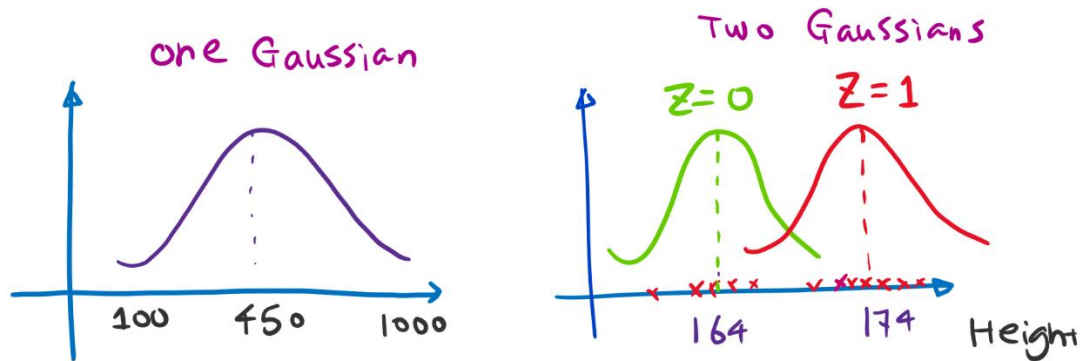


Gaussian Mixture Model (GMM)

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$$P(x|\theta) \sim N(\mu, \delta) \quad (\text{Gaussian PDF})$$

$$P(x|\theta) = P(x|\mu_1, \mu_2, \delta_1, \delta_2, \pi) \quad \text{Mixture of two Gaussians}$$
$$= \pi N(\mu_1, \delta_1) + (1-\pi) N(\mu_2, \delta_2)$$

Given training data $D = \{x^{(1)}, x^{(2)}, \dots, x^{(N)}\}$

$$L(\theta) = \arg \max_{\theta} P(D|\theta)$$
$$= \arg \max_{\theta} \prod_{i=1}^N P(x^{(i)}|\theta)$$
$$= \arg \max_{\theta} \sum_{i=1}^N \log P(x^{(i)}|\theta)$$

likelihood of D
given parameters θ

$$\begin{aligned}
P(x|\theta) &= \sum_z P(x, z|\theta) \\
&= \sum_z P(x|z, \theta) P(z|\theta) \quad z = \{0, 1\} \\
&= \sum_z P(x|z=0, \theta) P(z=0|\theta) + P(x|z=1, \theta) P(z=1|\theta) \\
&= \pi N(\mu_1, \delta_1) + (1-\pi) N(\mu_2, \delta_2)
\end{aligned}$$

$$\text{E-step: } w^{(j)} = \frac{\pi P(x^{(j)}|\mu_1, \delta_1)}{\pi P(x^{(j)}|\mu_1, \delta_1) + (1-\pi) P(x^{(j)}|\mu_2, \delta_2)}$$

$$\text{M-step: } \pi = \frac{\sum_{j=1}^N w^{(j)}}{N}$$

$$\mu_1 = \frac{\sum_j w^{(j)} x^{(j)}}{\sum_{j=1}^N w^{(j)}}$$

$$\mu_2 = \frac{\sum_j (1-w^{(j)}) x^{(j)}}{\sum_{j=1}^N (1-w^{(j)})}$$

$$\delta_1^2 = \frac{w^{(j)} (x^{(j)} - \mu_1)^2}{\sum_{j=1}^N w^{(j)}}$$

$$\delta_2^2 = \frac{(1-w^{(j)}) (x^{(j)} - \mu_2)^2}{\sum_{j=1}^N (1-w^{(j)})}$$